graphing the black body and visual response spectra

$T_{color} \equiv 2856 \cdot K$ color temperature of source used for CCD measurement

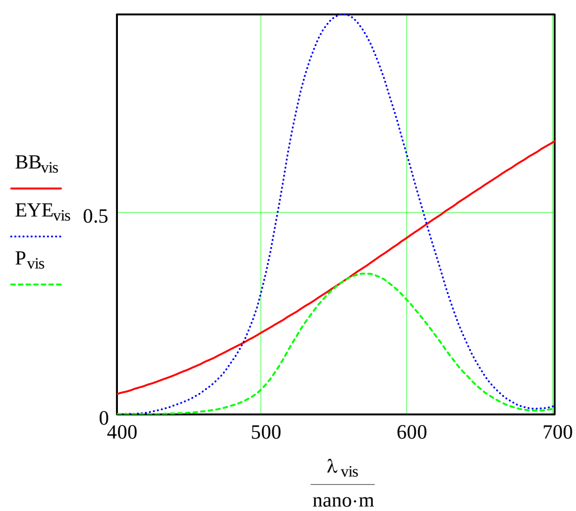
$vis := 0..100$ $E\Lambda_{max} := 700 \cdot nano \cdot m$ $E\Lambda_{min} := 400 \cdot nano \cdot m$

$$\lambda_{vis} := E\Lambda_{min} + \left(E\Lambda_{max} - E\Lambda_{min}\right) \cdot \frac{vis}{100}$$

$BB_{vis} := BB_{\lambda}\left(\lambda_{vis}, T_{color}\right)$ normalized black body

$EYE_{vis} := VIS\left(\lambda_{vis}\right)$ photopic vision

$P_{vis} := BB_{vis} \cdot EYE_{vis}$ product of the above



red : black body
blue : normalized photopic eye response
green : eye response to black body spectrum

$$VRI := \int_{E\Lambda_{min}}^{E\Lambda_{max}} BB_{\lambda}\left(\Lambda, T_{color}\right) \cdot VIS(\Lambda)\, d\Lambda \qquad VRI = 3.724 \times 10^{-8}\, m$$

VRI
integral of
eye response to black body spectrum

$S_v := 100 \cdot lx$ illumination on imager : specified in imager data sheet

$$K_E := \frac{S_v}{K_P \cdot VRI} \qquad K_E = 3.99\, W \cdot m^{-2} \cdot micron^{-1}$$

$$M_{peak}\left(T_{color}\right) = 2.448 \times 10^{6}\, W \cdot m^{-2} \cdot micron^{-1}$$

graphing the detector response

$D\Lambda_{min} := 200 \cdot nano \cdot m \quad D\Lambda_{max} := 1100 \cdot nano \cdot m$ wavelength range

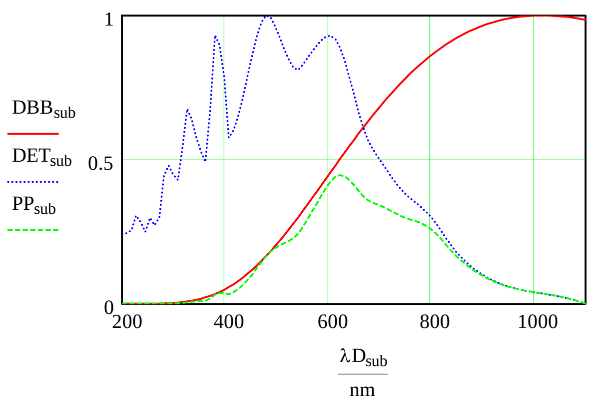
$sub := 0 .. 100$ calculation control subscript

$\lambda D_{sub} := D\Lambda_{min} + \left(D\Lambda_{max} - D\Lambda_{min}\right) \cdot \frac{sub}{100}$ wavelenght of subth calculation

$DET_{sub} := DSR\left(\lambda D_{sub}\right)$ detector spectral response

$DBB_{sub} := BB_{\lambda}\left(\lambda D_{sub}, T_{color}\right)$ black body spectrum

$PP_{sub} := DBB_{sub} \cdot DET_{sub}$ detector spectral response to black body excitation



red : black body
blue : detector spectral response
green : detector response to black body spectrum

calculate detector experimental responses

$DE := \int_{D\Lambda_{min}}^{D\Lambda_{max}} BB_{\lambda}\left(\Lambda, T_{color}\right) \cdot DSR(\Lambda)\, d\Lambda \qquad DE = 1.358 \times 10^{-7} m$ DE detector effectivity

$DSI(\lambda) := K_E \cdot BB_{\lambda}\left(\lambda, T_{color}\right)$ detector spectral irradiance

$DI := \int_{D\Lambda_{min}}^{D\Lambda_{max}} DSI(\Lambda)\, d\Lambda \qquad DI = 1.879\, W \cdot m^{-2}$ detector irradiance

$PSI(\lambda) := \frac{DSI(\lambda)}{\left(\frac{h \cdot c}{\lambda}\right)}$ function for detector spectral photon irradiance

$PI := \int_{D\Lambda_{min}}^{D\Lambda_{max}} PSI(\Lambda)\, d\Lambda \qquad PI = 7.961 \times 10^{18} \frac{1}{m^2 s}$ detector photon irradiance

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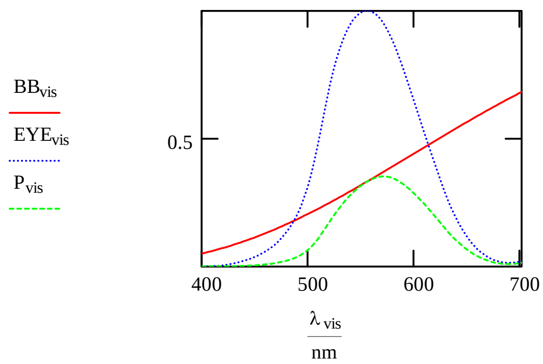

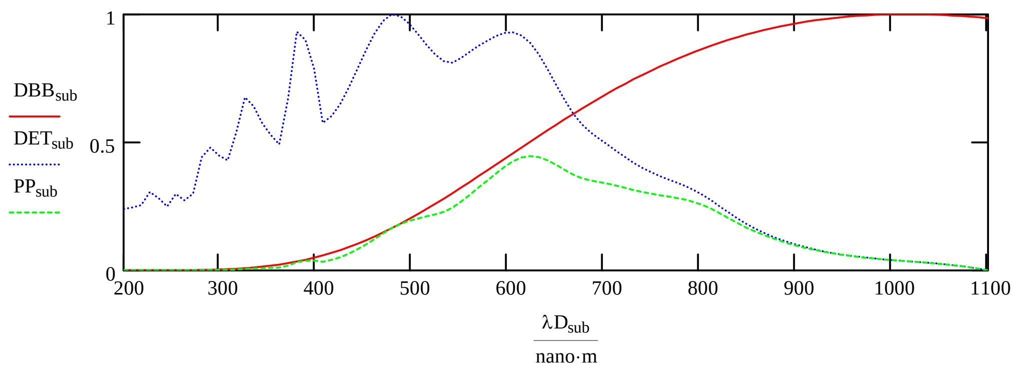

detector geometry

$H_{pixel} := 0.50 \cdot mm$ $W_{pixel} := 25 \cdot micron$ $A_{pixel} := H_{pixel} \cdot W_{pixel}$ $A_{pixel} = 1.25 \times 10^{-4} cm^2$

$PI \cdot A_{pixel} = 9.951 \times 10^{10} s^{-1}$ photon flux on a pixel

$EPSI(\lambda) := PSI(\lambda) \cdot DSR(\lambda)$ effective detector spectral photon irradiance

$EPI := \int_{D\Lambda_{min}}^{D\Lambda_{max}} A_{pixel} EPSI(\Lambda)\, d\Lambda$ effective detector photon flux these are the photons that are detected $EPI = 2.311 \times 10^{10} s^{-1}$

$E_{sat} := 0.145 \cdot lx \cdot s$ saturation exposure

$t_{sat} := \frac{E_{sat}}{S_v}$ corresponding exposure time $t_{sat} = 1.45 \times 10^{-3} s$

$SPE := EPI \cdot t_{sat}$ saturation photon exposure $SPE = 3.351 \times 10^{7}$

$SEC := SPE \cdot q$ saturation charge on pixel $SEC = 5.369 \times 10^{-12} C$

Professor Jim Palmers Radiometry FAQ page University of Arizona
http://www.optics.arizona.edu/Palmer/rpfaq/rpfaq.htm

constants and conversions

$h \equiv 6.626 \cdot 10^{-34} J \cdot s$ Planck's constant

$k \equiv 1.381 \cdot 10^{-23} \cdot J \cdot K^{-1}$ Boltzmann's constant

$q \equiv 1.602 \cdot 10^{-19} \cdot C$ electron charge

$eV \equiv 1.602 \cdot 10^{-19} \cdot J$ electron volt

$c \equiv 2.998 \cdot 10^{8} \cdot m \cdot sec^{-1}$ speed of light

$\sigma \equiv 5.670 \cdot 10^{-8} \cdot W \cdot m^{-2} \cdot K^{-4}$ Stefan-Boltzmann constant

$K_P \equiv 673 \cdot lm \cdot W^{-1}$ photopic luminous efficacy @ 555 nm

$milli \equiv 10^{-3}$ $micro \equiv 10^{-6}$ $nano \equiv 10^{-9}$ $pico \equiv 10^{-12}$

$micron \equiv micro \cdot m$ $nm \equiv m \cdot 10^{-9}$

GLOBAL DEFINITIONS OF FUNCTIONS

black body spectral radiant exitance

$$M_{\lambda}(\lambda,T) \equiv \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5\cdot\left(e^{\frac{h\cdot c}{\lambda\cdot k\cdot T}} - 1\right)}$$

$Typical\_Units := W\cdot m^{-2}\cdot micron^{-1}$

Wein's displacement law

$$\Lambda_{peak}(T) \equiv \frac{2898}{T}\cdot micro\cdot m\cdot K$$

This gives the wavelength of the peak of the black body spectrum at temperture T

$Typical\_Units := micron$

spectral radiant excitance at black body peak

This gives the spectral radiant excitance of the peak of the black body spectrum at temperature T

$$M_{peak}(T) \equiv M_{\lambda}\left(\Lambda_{peak}(T), T\right)$$

$Typical\_Units := W\cdot m^{-2}$

normalized black body

This gives the spectrum, normalized to a peak of 1, of a black body at temperature T

$$BB_{\lambda}(\lambda,T) \equiv \frac{M_{\lambda}(\lambda,T)}{M_{peak}(T)}$$

$Units := dimensionless$

black body total radiant excitance

This gives the total radiant power emitted from a black body at temperatute T.

$$BBP(T) \equiv \sigma\cdot T^4$$

$Typical\_Units := W\cdot m^{-2}$

radiant exitance

This gives the radiant excitance over a passband
center wavelength = λ
band width $= \Delta\lambda$
for a black body spectrum at temperature T

$$M(\lambda,\Delta\lambda,T) \equiv \left(\int_{\lambda-\frac{\Delta\lambda}{2}}^{\lambda+\frac{\Delta\lambda}{2}} M_{\lambda}(\lambda,T)\,d\lambda\right)$$

$Typical\_Units := W\cdot m^{-2}$

GLOBAL DEFINITION OF PHOTOPIC VISION CURVE

$$\text{lamda} \equiv \begin{pmatrix} 400 \\ 410 \\ 420 \\ 430 \\ 440 \\ 450 \\ 460 \\ 470 \\ 480 \\ 490 \\ 500 \\ 510 \\ 520 \\ 530 \\ 540 \\ 550 \\ 560 \\ 570 \\ 580 \\ 590 \\ 600 \\ 610 \\ 620 \\ 630 \\ 640 \\ 650 \\ 660 \\ 670 \\ 680 \end{pmatrix} \cdot 10^{-9} \cdot \text{m} \qquad \text{photopic} \equiv \begin{pmatrix} .0004 \\ .0012 \\ .0040 \\ .0116 \\ .0230 \\ .0380 \\ .0600 \\ .0910 \\ .1390 \\ .2080 \\ .3230 \\ .503 \\ .7100 \\ .8620 \\ .9540 \\ .9950 \\ .9950 \\ .9520 \\ .8700 \\ .7570 \\ .6310 \\ .5030 \\ .3810 \\ .2650 \\ .1750 \\ .1070 \\ .0610 \\ .0320 \\ .0170 \end{pmatrix}$$

$\text{vision} \equiv \text{cspline}(\text{lamda}, \text{photopic})$

$\text{VIS}(\text{color}) \equiv \text{interp}(\text{vision}, \text{lamda}, \text{photopic}, \text{color})$

$\text{VIS}(500 \cdot \text{nano} \cdot \text{m}) = 0.323$

GLOBAL DEFINITION OF DETECTOR SPECTRAL RESPONSE CURVE

DETECTOR Hamamatsu S8377/S8378 series

$$\Lambda \equiv \begin{pmatrix} 200 \\ 205 \\ 225 \\ 230 \\ 235 \\ 241 \\ 243 \\ 245 \\ 247 \\ 250 \\ 255 \\ 260 \\ 265 \\ 270 \\ 283 \\ 290 \\ 295 \\ 302 \\ 308 \\ 314 \\ 320 \\ 327 \\ 334 \\ 352 \\ 360 \\ 368 \\ 375 \\ 385 \\ 395 \\ 403 \\ 410 \\ 420 \end{pmatrix} \cdot nm \qquad R \equiv \begin{pmatrix} 0.24 \\ 0.245 \\ 0.29 \\ 0.32 \\ 0.29 \\ 0.25 \\ 0.24 \\ 0.25 \\ 0.295 \\ 0.31 \\ 0.295 \\ 0.28 \\ 0.27 \\ 0.28 \\ 0.465 \\ 0.48 \\ 0.465 \\ 0.44 \\ 0.43 \\ 0.44 \\ 0.645 \\ 0.67 \\ 0.645 \\ 0.53 \\ 0.51 \\ 0.53 \\ 0.875 \\ 0.89 \\ 0.875 \\ 0.62 \\ 0.58 \\ 0.62 \end{pmatrix}$$

465	0.95
480	1
500	0.95
525	0.84
540	0.81
555	0.84
580	0.9
600	0.93
620	0.9
670	0.6
700	0.5
740	0.4
800	0.3
840	0.2
900	0.1
1000	0.04
1100	0

interpolation functions for the detector spectral response curve

$$\mathrm{Ham} \equiv \mathrm{cspline}(\Lambda, R)$$

$$\mathrm{DSR}(\mathrm{color}) \equiv \mathrm{interp}(\mathrm{Ham}, \Lambda, R, \mathrm{color})$$ detector spectral response