This model is a test of calculations the normalized black body and visual response distributions and the physical constants used to provide physically meaningful results. The curve on page 2 will be compared with a curve in the RCA Electro-Optics Handbook (pg 41), they should be identical if the algebra is correct.

For calculations such as the transmissions of several components in series the peaks of the band pass curves may be normalized to 1, then the shape of the transmission function is given by the product of the bandpass curves. When the calculation involves black body radiation, the shape of the excitation curve as well as the amplitude change with temperature. The result of this calculation is the "illumination efficacy and efficiency" of a black body as a function of temperature. It is necessary to account for the amplitude change with an equation that describes that change, as well as the normalizes black body equation.

set up of the calculations

$E\Lambda_{max} := 680 \cdot nano \cdot m \qquad E\Lambda_{min} := 400 \cdot nano \cdot m$ range of visable wavelengths

$T_{max} := 20 \cdot 10^3 \cdot K \qquad T_{min} := 2 \cdot 10^3 \cdot K$ the range of color temperatures

$N := 50$ number of calculations - 1

$R := 10^{\frac{1}{N} \cdot \log\left(\frac{T_{max}}{T_{min}}\right)} \qquad R = 0.556\,K$ the spacing between adjacent color temperatures is geometric, R is the ratio

$count := 0 .. N$ calculation counter

$T_{count} := T_{min} \cdot R^{count}$ color temperature for the calculation

calculations of luminous efficacy and luminous efficiency

$$PE_{count} := \int_{E\Lambda_{min}}^{E\Lambda_{max}} BB_{\lambda}\left(\Lambda, T_{count}\right) \cdot VIS(\Lambda)\, d\Lambda$$

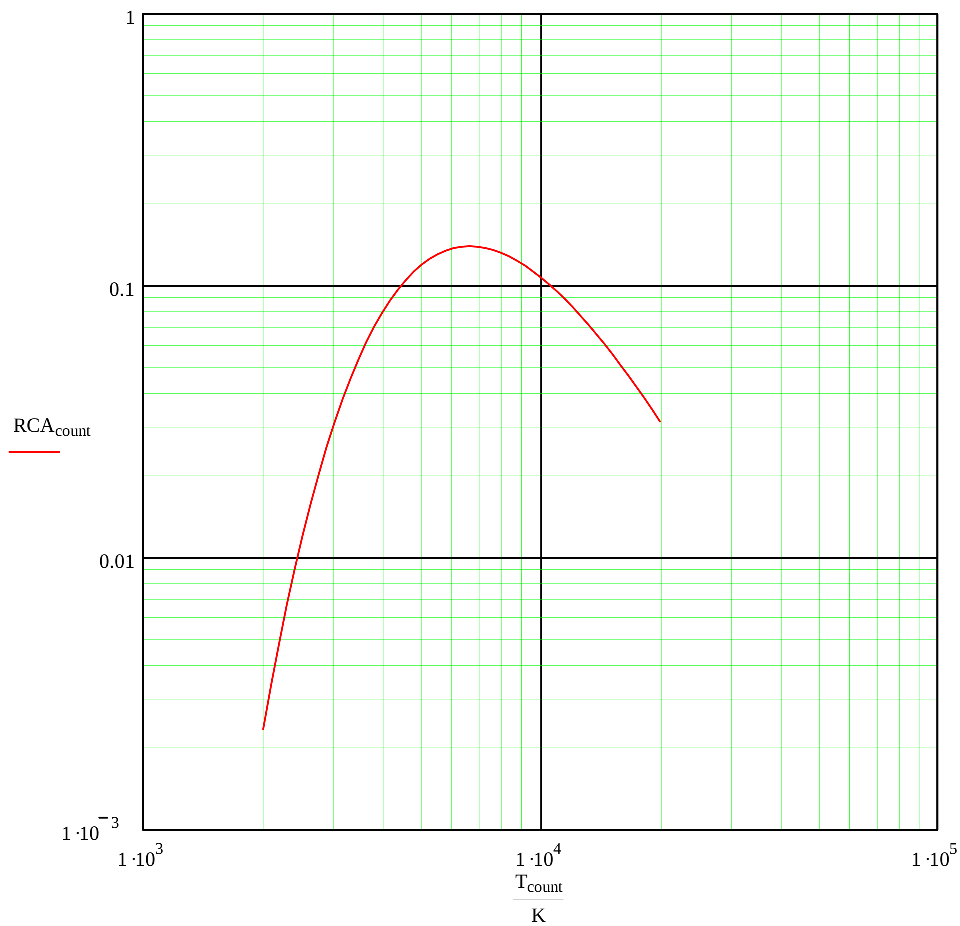
Product of normalized black body distribution for T=T.count and visual response. This equation only incorporates the shape change of the black body spectrum with temperature.

$Typical\_Units := microns$

$$RCA_{count} := \frac{M_{peak}\left(T_{count}\right)}{\sigma \cdot \left(T_{count}\right)^4} \cdot PE_{count}$$

The quotient is the ratio of the peak spectral radiance to the total radiance. The peak spectral radiance is the quantity needed to account for the amplitude change of the black body curve. The total radiance in the denominator makes this a power efficiency quantity.

Plot of luminous efficiency versus color temperature for a perfect black body source



$T_{color} := 6600 \cdot K$ the peak color temperature estimated from the graph

$$best := K_L \cdot \frac{M_{peak}(T_{color})}{\sigma \cdot T_{color}^{\;4}} \cdot \int_{E\Lambda_{min}}^{E\Lambda_{max}} BB_\lambda(\Lambda, T_{color}) \cdot VIS(\Lambda)\, d\Lambda \qquad best = 93.967 \frac{lm}{W}$$

the RCA handbook gives 95 lm/W for the quantity I call best

$\Lambda_{peak}(T_{color}) = 0.439\, micro \cdot m$

This is not the peak of the visual response curve, demonstrating the need to account for both black body curve shape and amplitude for calculations similar to this.

constants and conversions

$h \equiv 6.626\cdot 10^{-34} J\cdot s$ Planck's constant

$k \equiv 1.381\cdot 10^{-23}\cdot J\cdot K^{-1}$ Boltzmann's constant

$q \equiv 1.602\cdot 10^{-19}\cdot C$ electron charge

$eV \equiv 1.602\cdot 10^{-19}\cdot J$ electron volt

$c \equiv 2.998\cdot 10^{8}\cdot m\cdot sec^{-1}$ speed of light

$\sigma \equiv 5.670\cdot 10^{-8}\cdot W\cdot m^{-2}\cdot K^{-4}$ Stefan-Boltzmann constant

$K_L \equiv 673\cdot lm\cdot W^{-1}$ luminous efficacy @ 555 nano meters

$milli \equiv 10^{-3}$ $micro \equiv 10^{-6}$ $nano \equiv 10^{-9}$ $pico \equiv 10^{-12}$

$micron \equiv micro\cdot m$ $nm \equiv m\cdot 10^{-9}$

GLOBAL DEFINITIONS OF BLACK BODY FUNCTIONS

spectral radiant exitance

$$M_\lambda(\lambda,T) \equiv \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5\cdot\left(e^{\frac{h\cdot c}{\lambda\cdot k\cdot T}} - 1\right)}$$

$$Typical\_Units := W\cdot m^{-2}\cdot micron^{-1}$$

Wein's displacement law

This gives the wavelength of the peak
of the black body spectrum at temperature T

$$\Lambda_{peak}(T) \equiv \frac{2898}{T}\cdot micro\cdot m\cdot K$$

$$Typical\_Units := micron$$

spectral radiant exitance at black body peak

This gives the spectral radiant excitance of the peak of the black body spectrum at temperature T

$$M_{peak}(T) \equiv M_\lambda\left(\Lambda_{peak}(T),T\right)$$

$$Typical\_Units := W\cdot m^{-2}$$

normalized black body

This gives the spectrum, normalized to a peak of 1, of a black body at temperature T

$$BB_\lambda(\lambda,T) \equiv \frac{M_\lambda(\lambda,T)}{M_{peak}(T)}$$

$$Units := dimensionless$$

black body total radiant excitance

This gives the total radiant power emitted from a black body at temperature T.
The proof is left as an exercise for the engineer with too much time on his/her hands.

$$BBP(T) \equiv \sigma\cdot T^4$$

$$Typical\_Units := W\cdot m^{-2}$$

radiant exitance

$$M(\lambda,\Delta\lambda,T) \equiv \left(\int_{\lambda-\frac{\Delta\lambda}{2}}^{\lambda+\frac{\Delta\lambda}{2}} M_\lambda(\lambda,T)\,d\lambda\right)$$

This gives the radiant excitance over a passband with
center wavelength = λ
band width = $\Delta\lambda$
for a black body spectrum at temperature T

GLOBAL DEFINITION OF PHOTOPIC VISION CURVE

$$\text{lamda} \equiv \begin{pmatrix} 400 \\ 410 \\ 420 \\ 430 \\ 440 \\ 450 \\ 460 \\ 470 \\ 480 \\ 490 \\ 500 \\ 510 \\ 520 \\ 530 \\ 540 \\ 550 \\ 560 \\ 570 \\ 580 \\ 590 \\ 600 \\ 610 \\ 620 \\ 630 \\ 640 \\ 650 \\ 660 \\ 670 \\ 680 \end{pmatrix} \cdot 10^{-9} \cdot \text{m} \qquad \text{photopic} \equiv \begin{pmatrix} .0004 \\ .0012 \\ .0040 \\ .0116 \\ .0230 \\ .0380 \\ .0600 \\ .0910 \\ .1390 \\ .2080 \\ .3230 \\ .503 \\ .7100 \\ .8620 \\ .9540 \\ .9950 \\ .9950 \\ .9520 \\ .8700 \\ .7570 \\ .6310 \\ .5030 \\ .3810 \\ .2650 \\ .1750 \\ .1070 \\ .0610 \\ .0320 \\ .0170 \end{pmatrix}$$

$$\text{vision} \equiv \text{cspline}(\text{lamda}, \text{photopic})$$

$$\text{VIS}(\text{color}) \equiv \text{interp}(\text{vision}, \text{lamda}, \text{photopic}, \text{color})$$

$$\text{VIS}(500 \cdot \text{nano} \cdot \text{m}) = 0.323$$