# Interferometric Measurement of Rotationally Symmetric Aspheric Surfaces 

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#### Abstract

A new method for measuring the surface of aspheric optics using a combination of two interferometric technologies is presented. The metrology method provides a 3D measurement with high data density in a short measurement time without the need for special tooling. The measurement technique is inherently insensitive to ray trace error and can be used on the shop floor. It covers a large range of aspheric departures and delivers very small measurement uncertainties.


Keywords: Aspheric Testing, Absolute Testing, Fizeau Interferometry, Optical Metrology, Aspheric Surfaces, Optical Fabrication, Optical Testing, Optical Design and Fabrication, Aspheric Lens Design.

## 1. INTRODUCTION

In the digital age, aspheric surfaces have become inevitable in modern optical lens designs. Aspheric surface can be found in pick-up lenses for CD, DVD and Blue Ray Discs, in cameras built into mobile phones, in zoom-lenses for small digital cameras and camcorders, digital SLRs and Television Cameras, and especially, for lithographic projection lenses. The range of surface diameters spans more than two orders of magnitude for refractive optics, and the aspheric departure can be a few microns or as large as 5 mm . The required uncertainty of the measurements can be as small as 0.1 nm r.m.s., and the required spatial resolution as high as 2000 points across the diameter of the lens. Without a means to measure these surfaces, they cannot be manufactured!

As the lenses are used to "shape" light, it is very natural also to use light for measuring of the shape of the surfaces produced. But the great beauty of spherical Fizeau interferometry that one transmission sphere can be used to test a large variety of spherical lenses by simply adjusting the test set-up for the radius of the part is no longer valid. To be measured at one stroke, every aspheric surface design would need its own, unique reference to be compared with. In order to preserve all the advantages of an optical measurement, being non-contact, fast and very high spatial resolution as well as very precise, unique references for each asphere shape have been used. These unique references are either diffractive ${ }^{1}$ or refractive ${ }^{2}$ and adapt the wavefront of a transmission reference to the shape of the aspheric test surface. This method of "null compensation" has great disadvantages compared to interferometry on spherical surfaces; the null compensator "tools" have a long manufacturing cycle time and they must be somehow qualified which is itself a difficult metrology problem.

Stitching methods have been developed ${ }^{3}$ that piece together subsections of the aspheric surface that have been measured relative to a spherical reference surface; each stitching subsection is measured at a different orientation. As the movement of the aspheric surface in 5 degrees of freedom can not be measured to the needed precision, this information is derived from the measurement result in overlapping subsections on the surface. By its very nature, these overlapping regions become smaller with increasing aspheric departure and at the same time the system inherent errors of higher order, the so-called retrace errors for non-null tests, become more severe. Even applying highly sophisticated calibration strategies, these effects limit the applicability of this method for parts with higher aspheric departure.

To overcome all the problems associated with adapting the tool to the surface to be measured in a radical manner, people have given up the parallelism of optical measurement totally and developed high precision coordinate measurement machines with a mechanical probe touching the surface. Now the surface is measured point by point, and in addition in a $x, y, z$ coordinate system. As a consequence, the "dynamic range" of the metrological problem has become very high, and due to its sequential nature, the result can be subject to drift problems. In addition, only a limited number of points can be measured in reasonable time, so the spatial bandwidth of the measurement result is limited. Coordinate measuring machines inherent features make them very difficult to design for production use.

## 2. THE NEW METHOD ${ }^{4}$

Analyzing the problem of aspheric surfaces, it was noticed that for a presentation of the surface in a cylinder coordinate system, $\mathrm{z}=\mathrm{z}(\mathrm{h}, \theta)$ (with $\mathrm{z}=$ axial coordinate, $\mathrm{h}=$ lateral coordinate, and $\theta=$ azimuthal angle) the ideal surface does not change with $\theta$. But this means that a rotationally aspheric surface can be treated as a problem with one independent and one dependent variable, i.e. as a problem with two variables only, $h$ and $z$. Further it was noticed, that an aspheric surface can be completely tested against a spherical reference surface, when the surface is scanned along its symmetry axis, the z-axis. When scanned, we get resolvable interference fringes at zones, where the aspheric surface and the spherical reference surface have common tangents, i.e. where the rays are normal to both surfaces. But at those areas, we also have the ideal metrology conditions, the so-called "null-test" conditions, where the rays from both surfaces are common path through all optics and therefore no retrace errors are induced in the interferometer system. Thus, by simply scanning along the symmetry axis, we can collect interferometric measurements from the complete surface, see figure 1.


Figure 1: Asphere Interferometer Measurement Schematic: when the aspheric surface is scanned along the symmetry axis two areas show the "null-test" conditions at the same time: the center and a "zone".

But this alone does not solve the problem of measuring the aspheric surface: now we have a collection of ringshaped areas measured on the given surface, but this collection does not yet describe surface deviations from design. Another idea has to be added to tie together all these rings as if they were measured all at the same time in the common path condition. This idea is to measure a quantity on the aspheric surface which characterizes uniquely its shape and is "macroscopically" dependent but "microscopically" independent from the relative orientation of the reference surface to the test surface. This quantity is the difference in the distance of the apex and of the "zone" of the asphere from the spherical reference surface as a function of the scan position, see figure 2 . From this difference and the scan position we can "reconstruct" the surface point by point. No "overlap" of measured areas is needed, i.e. every measurement is independent from the others. What we use as the important information are relative distance measurements between two solid bodies, the test surface and the reference surface! This is very different from the idea of stitching and leads to highest accuracy in the result!


Figure 2: The aspheric surface is scanned along the symmetry axis; at every scan position, the quantities $\mathrm{v}_{\mathrm{n}}=\mathrm{d}_{\text {confocal- }} \mathrm{d}_{\text {apex_n }}$ and $\mathrm{p}_{\mathrm{n}}=\mathrm{d}_{\text {zone_ }} \mathrm{n}-\mathrm{d}_{\text {apex_n }}$ are measured.

The "sag" $z$ of the aspheric surface and its first derivative $d z / d h$ with respect to the independent variable $h$ are described by the equations (1) and (2):

$$
\begin{align*}
& z(h)=\left(\frac{c}{1+\sqrt{1-(1+k) c^{2} h^{2}}}+a_{2}\right) h^{2}+a_{3} h^{3}+a_{4} h^{4}+\ldots+a_{n} h^{n}  \tag{1}\\
& k=\text { conic constant, } \quad R_{0}=\frac{\sqrt{\left(1+z^{\prime}(0)^{2}\right)^{3}}}{z^{\prime \prime}(0)}=\frac{1}{c+2 a_{2}} \quad \text { radius of the apex-sphere } \\
& z^{\prime}(h)=\frac{c \cdot h}{\sqrt{1-(1+k) c^{2} h^{2}}}+2 a_{2} h+3 a_{3} h^{2}+4 a_{4} h^{3}+\ldots+n \cdot a_{n} h^{n-1} \tag{2}
\end{align*}
$$

It can be shown ${ }^{5}$, that the two quantities $v$ and $p$, which we measure by interferometric measurement techniques, can be converted to $h$ and $z$; thus they also uniquely describe the aspheric surface, see equations (3), (4) and (5).

$$
\begin{align*}
& v=z-R_{0}+\frac{h}{z^{\prime}}  \tag{3a}\\
& p=z+\frac{1-\sqrt{1+z^{\prime 2}}}{z^{\prime}} \cdot h  \tag{4a}\\
& \frac{d p}{d v}=p^{\prime}=1-\frac{1}{\sqrt{1+z^{\prime 2}}} \tag{5a}
\end{align*}
$$

$$
\begin{align*}
& h=\left(R_{0}+v-p\right) \sqrt{p^{\prime}\left(2-p^{\prime}\right)}  \tag{3b}\\
& z=p+\left(R_{0}+v-p\right) \cdot p^{\prime}  \tag{4b}\\
& \frac{d z}{d h}=z^{\prime}=\frac{\sqrt{p^{\prime}\left(2-p^{\prime}\right)}}{1-p^{\prime}} \tag{5b}
\end{align*}
$$

In addition, the quantity $\Delta \mathrm{p}(\mathrm{h}, \theta)$, which is the deviation of the measured surface from the design value, is automatically oriented normal to the surface. Please notice, that in order to convert p and v into h and z , the additional quantity $d p / d v \approx \Delta p / \Delta v$ is needed. This is the differential change of the value for $p$ by a differential change of the value for $v$. This quantity can either be deduced by differentiating the function $p=p(v)$ or directly measured. Notice that here for the first time in interferometric measurement technology, the lateral coordinate $h$ of the location of a shape deviation $\Delta \mathrm{p}(\mathrm{h}, \theta)$ is achieved from interferometric measurements, not from pixel coordinates; our coordinate system for the measurement is in object space! Due to eq. (3) to (5), neither the knowledge of the actual magnification is needed, nor can any distortion of the imaging system deteriorate the accuracy of the result. Before the scanning starts, the part is brought to "cat's eye position", where the apex of the part coincides with the center point of the reference surface. Then we change the position by the amount $\mathrm{R}_{0}{ }^{*}$ to reach the position shown in Fig. 2a. This way the distance $\mathrm{R}_{0}{ }^{*}$ from the apex of the surface to the center point M of the reference sphere (see Fig. 2) is known absolutely, i.e. the surface is measured including the deviation of the apex radius from the design value, or equivalently the "power" of the surface. This is a mayor difficulty when using interferometric compensation techniques, like null-lenses or holograms. Here, an error in the positioning of the aspheric surface relative to the null device influences largely the measured aspheric departure, especially on high NA surfaces.

## 3. THE INSTRUMENT

Zygo has developed the VeriFire ${ }^{\mathrm{TM}}$ Asphere (VFA) system ${ }^{6}$ as a powerful metrology tool for direct measurement of aspheric surfaces to be used in the production area. Departure from best-fit sphere approaching 800 microns and form error approaching 10 microns can be measured. This unique asphere technology combines two core Zygo competencies: 1) phase-shift Fizeau interferometry for the measurement of $p$ and 2) displacement interferometry for the measurement of v , tied together by proprietary software. Zygo's MetroCell ${ }^{\mathrm{TM}}$ environment isolation chamber is uniquely combined with a VeriFire ${ }^{\mathrm{TM}}$ AT interferometer using a 1 k x 1 k camera and a model 501 displacement measuring interferometer to form the VFA system, see Fig. 3; footprint is roughly $1 \mathrm{~m}^{2}$. Measurable part diameters can range from approximately 1 mm to 130 mm , and surface resolution will in most cases be greater than the part diameter divided by 900 . The typical TACT (Total Average Cycle Time) is less than 10 min and typically $>700,000$ points have then be measured on the surface. As the measurement is based on first principles described above, when an absolutely calibrated transmission sphere is used as the Fizeau reference surface, the aspheric measurement is "absolute" in the sense this term is used in our discipline.


Figure 3: VeriFire ${ }^{\mathrm{TM}}$ Asphere (VFA) system.


Figure 4: Typical measurement result

## 4. REFERENCES

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